Multiple Indicator Kriging Models in MineSight 3D

This is an introduction to Multiple Indicator Kriging (MIK) and how to prepare an MIK model within MineSight.

Why do we need MIK?

MIK is used to estimate the distribution of grades in a panel, or in a block (for local recovery estimation), or to estimate the probability of exceeding a certain threshold value (as in environmental problems).

For example, in the exploration phase of an ore deposit, an estimate of the global distribution provides a rough idea of the total tonnage of ore and quantity of metal above various cutoffs. In the feasibility and development phases, these global estimates are no longer sufficient. For long and short range planning you typically need estimates (distribution) of tonnage of ore and quantity of metal for smaller (local) block/areas.

MIK is also needed to handle estimation problems associated with highly skewed data, such as for gold deposits or hydrology. Linear methods are appropriate for estimating mean values; however, they may encounter problems calculating recoverable tonnages in the cases when the distribution of samples is highly skewed. For example, smoothing of high grade is one of the shortcomings of linear geostatistics like ordinary kriging (OK).

Using an ordinary variogram is often infeasible when the data are highly skewed, thereby making it impossible to use ordinary kriging (OK).

The rule of thumb to determine whether to use non-linear methods or not (MIK or OK) is that the coefficient of variation (CV) of the drillhole data is greater than 1.5. If CV is less than 0.5, linear methods will work. If greater than 1.5, they will not. If it's between 0.5 and 1.5, use linear interpolation with caution.

What is MIK?

MIK is a non-linear, non- parametric interpolation method. It is called non-linear because there are non-linear transformations involved (indicator function). It is called non-parametric because it does not make any assumptions about the distribution of the data.

MIK is based on the Indicator function

$$i(x,zc) = \begin{cases} 1, \text{ if } z(x) \le zc \\ 0, \text{ otherwise} \end{cases}$$

For a certain cutoff we can calculate the probability for the estimated block grade to be below the cutoff. For example:

32% probability CU ≤ 0.2

(68% probability CU > 0.2)

MIK applies indicator function to several cutoffs and estimates the local cumulative probability distribution of grades for a block.

For example, you have the following samples (Figure 1) and you want to calculate the grade in the block.

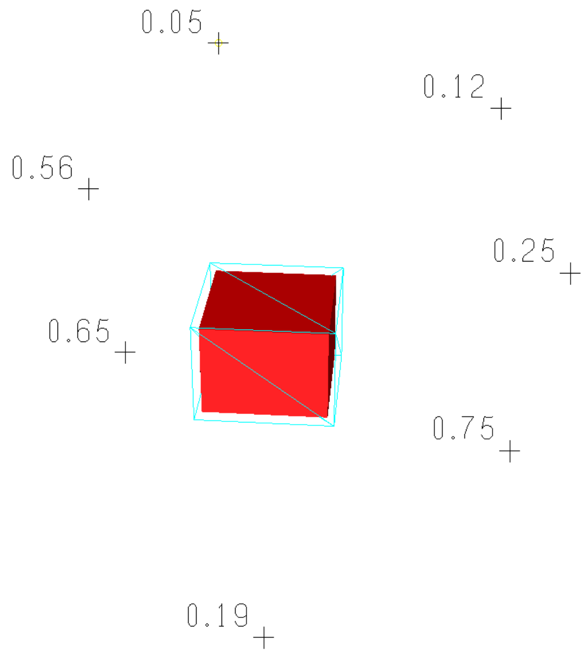

Figure 1. *An example of samples surrounding a block.*

Apply a series of cutoffs and convert the samples to indicators based on whether they are below or above the cutoff (Figure 2). In this example we have applied three arbitrary cutoffs to the eight sample values and converted the indicators.

Cutoff	Sample Value								
	0.05	0.12	0.19	0.25	0.56	0.65	0.75	0.91	
0.2	1	1	1	0	0	0	0	0	i(x;0.2)
0.5	1	1	1	1	0	0	0	0	i(x;0.5)
0.7	1	1	1	1	1	1	0	0	i(x:0.7)

Figure 2. *A series of cutoffs applied to the samples and converted to indicators.*

For mining applications, typically a few cutoff values have practical and economic significance. The mine plan may call for the separation of material in ore and waste based on a particular ore grade, or the ore material may be separated into a few stockpiles based on other cutoffs.

If no thresholds have special significance to the problems being estimated you can use the cutoffs that correspond to the nine deciles of the global distribution.

You could also run MineSight Data Analyst (MSDA) to calculate the cutoffs to be used for the MIK analysis. Based on a user specified number of cutoffs, MSDA tries to find the cutoffs that equalize the metal content within the bins (Figure 3).

Cutoff Analysis

File Properties

Cu comps_Rock 1-2 Ixp_Rock 1-2

Class	Cutoff >=	Cutoff <	Samples	Average	Metal (uni...	%Total
1	0.0	0.296	786	0.1462	114.93	9.0939
2	0.296	0.412	328	0.3511	115.15	9.1115
3	0.412	0.523	248	0.4634	114.93	9.0941
4	0.523	0.639	200	0.577	115.4	9.1311
5	0.639	0.746	166	0.6923	114.92	9.0934
6	0.746	0.857	145	0.7956	115.37	9.1287
7	0.857	0.987	126	0.9136	115.11	9.1085
8	0.987	1.152	109	1.0582	115.35	9.1271
9	1.152	1.393	90	1.2767	114.91	9.0921
10	1.393	1.669	77	1.5077	116.09	9.1861
11	1.669	3.609	57	1.9585	111.64	8.8333
Total:			2332	0.5419	1263.8	100.0

Figure 3. *Cutoff Analysis from MSDA.*

If there is a particular part of the distribution for which accurate estimation is more important, choose more cutoffs in that range. For example, in precious metals most of the metal is contained in a small proportion of very high grade. It makes sense in that case to perform indicator estimation at several high cutoffs since the estimation of the upper tail is more important than the estimation of the lower portion of the distribution.

For each series of indicators calculate the indicator variogram. You need to model the variograms. MSDA can be used to calculate and model the MIK variograms at the various cutoffs.

For example, for Cutoff = 0.2 model a spherical variogram (Figure 4):

Nugget	Sill	Y range	X range	Z range	R1	R2	R3
0.10	0.24	300	175	150	45	0	-5

Figure 4. *A spherical variogram for a single cutoff.*

Now krige the indicators (Figure 5).

Sample Value	i(x;0.2)	Kriging Weight	Cumulative Weights
0.05	1	0.12	0.12
0.12	1	0.10	0.22
0.19	1	0.10	0.32
0.25	0	0.21	0.53
0.56	0	0.21	0.74
0.65	0	0.00	0.74
0.75	0	0.15	0.89
0.91	0	0.11	1.00

Figure 5. *The kriged indicators.*

The probability of the block to be less than 0.2 is ip(x;0.2) = 32%

Repeat the exercise for the rest of the cutoffs.

Probability <0.5 =ip(x;0.5) = 50%

Probability <0.7 =ip(x;0.7) = 74%

Probability intervals for the second class: Prob{z(xo) Є(0.2,0.5)}=18%

Probability of exceeding cutoff: Prob{z(xo)>0.5 = 50%

The mean (E-type estimate): Z(xo) = Σclass frequency *class mean (Figure 6).

Class	<0.2	0.2-0.5	0.5-0.7	>0.7
Class mean	0.120	0.250	0.605	0.830
Frequency (%)	32	18	24	26

Figure 6. *The Class Mean and Frequency.*

Since we now have a "histogram" of probabilities for various cutoffs (Figure 7), we can even obtain local recoverable reserves at different cutoffs (different than the cutoffs we used to calculate the MIK probabilities).

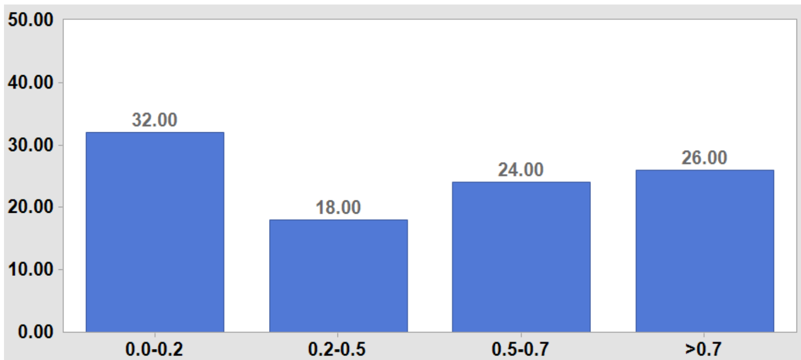

Figure 7. *Histogram of the probabilities for various cutoffs.*

For example(Figure 8):

CU > 0.0 : 100% @ 0.444

CU > 0.4 : 59% @ 0.650

CU > 0.6 : 38% @ 0.758

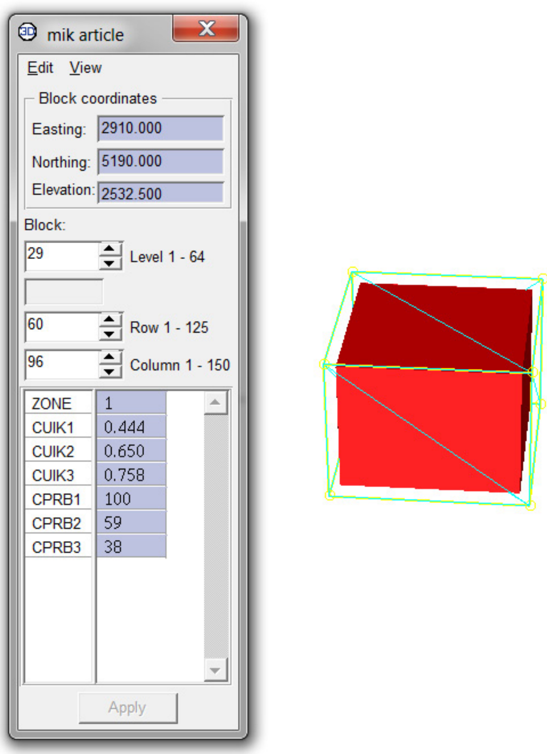

Figure 8. Local recoverable reserves of a block at different cutoffs.

MSBASIS program M624MIK performs the MIK calculations using procedure Multiple Indicator Kriging (MIK) (PMIK01.DAT), as shown in Figure 9.

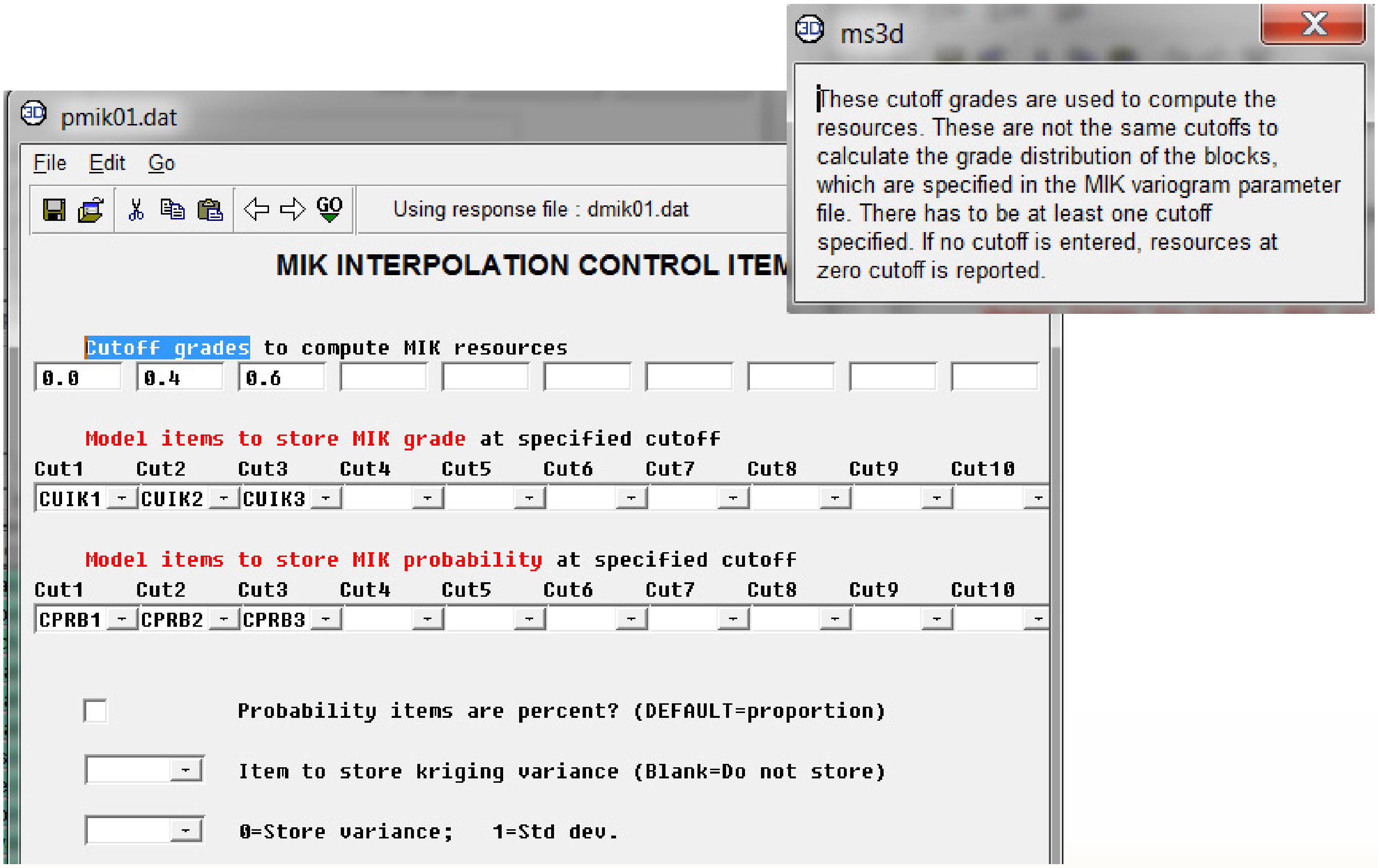

Figure 9. *Performing the actual MIK calculations with PMIK01.DAT.*

Conclusions

MIK can be a very useful approach to problems encountered by linear interpolation methods (especially in the case of highly skewed distributions). MIK can calculate grades and percentages above cutoffs. And MineSight provides a series of tools to create and manipulate MIK models.

Next month we will look at management of MIK models in MS3D.