MineSight® in the Foreground
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Kriging Variance and Other Error Related Output

The scope of this article is to present and explain some additional outputs (mainly error related) from the interpolation routines.

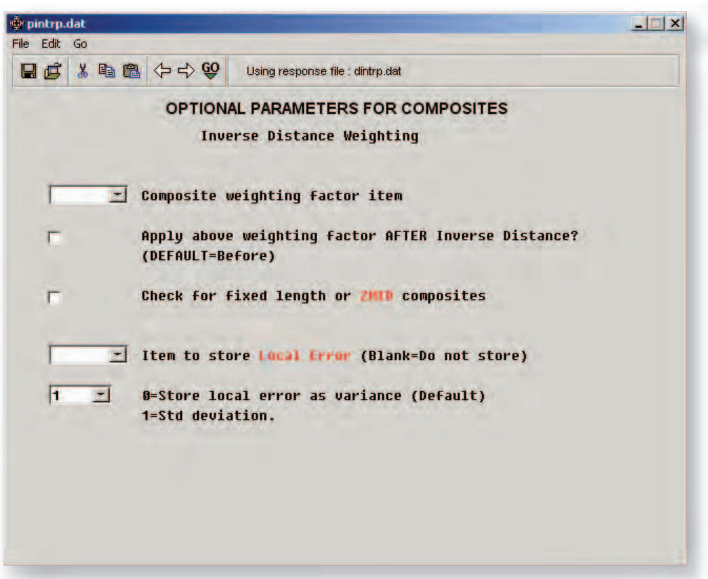
Inverse Distance Interpolation in MineSight® outputs the local error (or variance).

Kriging Interpolation in MineSight® outputs:

- Kriging Variance (or its Standard Deviation).
- Combined Variance (or its Standard Deviation).
- Relative Variance.
- Relative Variability Index.
- Local Variance divided by Kriging Variance.
- Sum of Kriging weights.
- Regression slope.

Multiple Indicator Kriging in MineSight® outputs the Conditional Variance.

Inverse Distance Weighting:



Local Error (or variance), in this case, is the weighted average of the differences between the block estimate and the data values:

$$\sigma^2_w = \Sigma\lambda^2_i * (Z_0 - Z_i)^2$$
$$i = 1, n \ (n>1)$$

where n is the number of data used,

λ_i are the weights corresponding to each datum,

Z_0 is the block estimate,

and Z_i are the data values.

Local Error (variance) should give you an idea of how similar your estimate is to the surrounding data. Local Error (variance) can also be used to compute Combined Variance (CVar). The CVar can be useful in identifying the areas that need additional drilling, to compute confidence intervals for the block grades and in resource classification. See discussion further on.

Standard Deviation of Local Error (option 1) can be stored instead of variance if calculated errors are very small (too many decimals).

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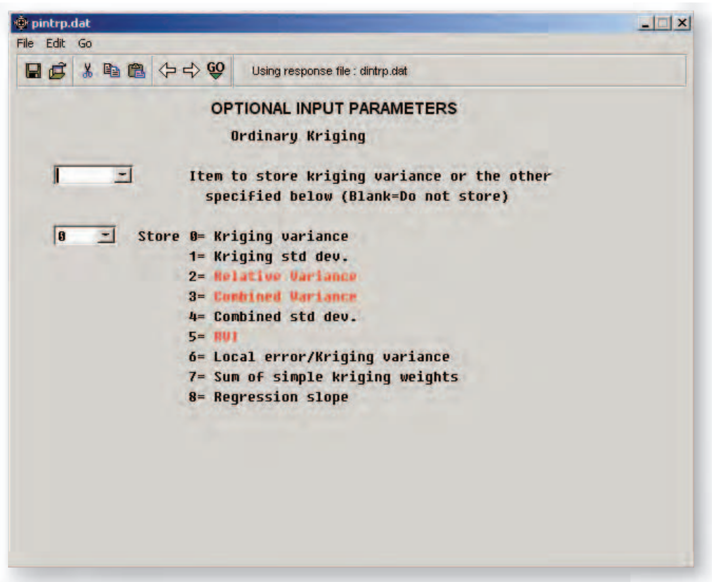
Kriging:



Kriging Variance is calculated by the following formula:

$$\sigma^2 = \sigma^2_Z + \Sigma\Sigma(\lambda_i \lambda_j C_{i,j}) - 2\,\Sigma\, \lambda_i C_{i,o}$$

where σ^2_Z is the sample variance,

$C_{i,j}$ is the covariance between samples,

and $C_{i,o}$ is the covariance between samples and the location of estimation.

This is the classical Kriging Variance calculation. In general the Kriging Variance will increase as variance of the data increases or as data becomes more redundant. Kriging Variance will decrease as data are closer to the location of estimation.

Kriging Standard Deviation is the square root of Kriging Variance. One may want to store the Standard Deviation (option 1) instead of variance if calculated errors are very small (too many decimals).

The Kriging Variance computed for a given point or block being estimated is essentially independent of the data values used in the estimation. It is purely a function of the spatial distribution and configuration of data points, and the version of the Kriging used. The only link between the Kriging Variance and data values is through the variogram, which is global rather than local in its definition. For this reason, the Kriging Variance does not always give an accurate reflection of the local variation. For instance, using the same variogram and the same data points around the block being estimated will give the same Kriging Variance, regardless of the values of the data points.

The Kriging Variance was not intended to specifically address the question of assessing uncertainty. It is an intermediate figure that is used to find an optimum set of weights. Although the Kriging Variance is a good measure of the spatial configuration of data points, it fails to impart their local variation since it does not depend on them directly. Therefore, any alternative measure of variance (see following options) should include a provision to measure the local variability.

Relative Variance is the Kriging Variance divided by the square of the Kriging Grade. See this as an attempt to normalize the Kriging Variance or to incorporate the actual grades.

Combined Variance has two components. One is the traditional Kriging Variance, the other is the local variance of the weighted average. With this approach of using combined variance, one essentially takes into account the local variation, as well as the spatial data configuration around the block being estimated.

It can be used in drillhole spacing studies and, in combination with the estimated grade, it can be used for reserves classification (see Relative Variability Index section below).

Combined Variance is the square root of the result of the Local Variance (σ^2_w) multiplied by Kriging Variance (σ^2_k):

$$CVar = SQ(\sigma^2_w \times \sigma^2_k)$$

where Local Variance (error) is calculated the same way as defined above in the Inverse Distance Weighting output section.

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Combined Standard Deviation is the square root of Combined Variance. One may need to store Standard Deviation (option 4) instead of variance if calculated errors are very small (too many decimals).

Relative Variability Index is the square root of Combined Variance divided by the Kriging Grade:

Relative Variability Index (RVI) =

$SQCVar/m_k \ (m_k > 0)$

where SQCVar is the square root of the Combined Variance, and m_k is the block grade computed from Kriging.

This index can be helpful in classifying the resources into measured, indicated and inferred categories based on the study of its distribution.

Further more, additional parameters can be incorporated into this index (each deposit may require different components). The calculation can also be customized for each deposit depending on the weight that must be given to certain measures. Use procedure **P61201.DAT** to perform model calculations.

Possible variables that can be incorporated into Relative Variability Index :

- Distance to the nearest sample from the centroid of the block,
- Actual number of composites used to interpolate the block,
- Number of quadrants with data (assuming a quadrant search is used) for a block,
- Number of octants with data (assuming an octant search is used),
- Number of diamond drillhole composites used for the block (assuming there are data quality problems with different types of drilling), etc.

Local Variance divided by Kriging Variance will give you an idea of how close the Kriging Variance is to the Local Variance (or how different it is). A low number (less than one) will indicate that Local Variance is lower than the Kriging Variance. This may mean that you have few available data (high Kriging Variance) but they are all similar.

If this number is greater than one, it would indicate that the Local Variance is higher than the Kriging Variance (you may have many data but they are very variable).

General note when using Kriging Variance:

If normalized variograms (or correlograms) are used, you may want to adjust the Kriging Variance by multiplying it by the variance of the data. In the case of Combined Variance, for example, CV is using the Local Variance of non-transformed data. Therefore, by multiplying the CV by the variance of the data, you convert the CV component of the normalized Kriging Variance to regular Kriging Variance. A model calculation is needed in this case (use pro cedure **P61201.DAT**).

Sum of Kriging weights is only available when one is using Simple Kriging. It is the summation of the Simple Kriging weights used for the sample data. Those Kriging weights do not add up to one; therefore, one may want to see how much weight is assigned to the mean ($Weight_{mean}$ = 1-sum of Kriging weights).

Regression slope is calculated by the following formula:

$$(\sigma^2_b - \sigma^2_k + LGM) \ / \ (\sigma^2_b - \sigma^2_k + 2 * LGM)$$

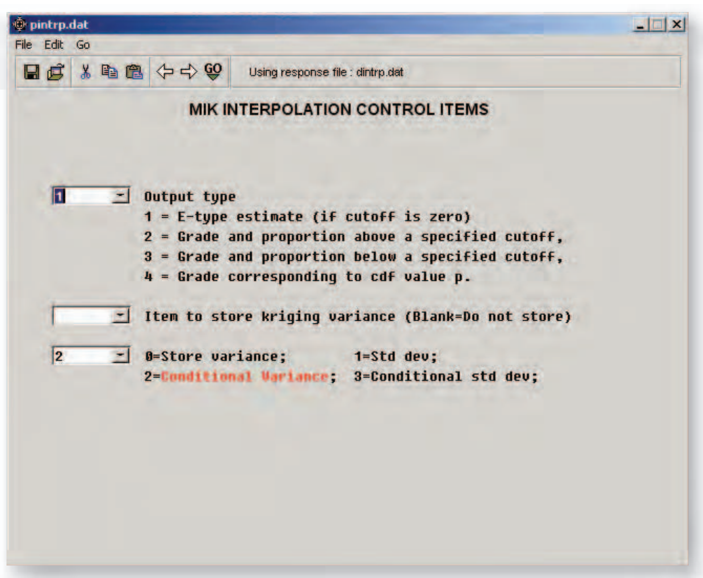
where σ^2_b is the block variance,

σ^2_k is the kriging variance,

and LGM is the LeGrange Multiplier.

One may use this figure to check whether or not the kriged grade of the block is conditionally unbiased. Blocks with less than 0.95 slope may be suspect.

Multiple Indicator Kriging:



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In MIK (using GSLIB routines), if E-type estimate is calculated, one has the option to store the Conditional Variance. Since this variance is conditional to the sample data, one overcomes the problem of data independence of the ordinary Kriging Variance.

It is calculated by the following formula:

Conditional Variance = e_{cv}/maxdis - etype*etype

where e_{cv} = sum of (z_{val}*z_{val}), etype = sum of (z_{val}), and z_{val} is the grade corresponding to each discretized cdf (cumulative density function) based on maxdis (# of discretizations).

References

- 30th APCOM International Symposium Proceedings, Phoenix, Arizona, 2002
  "Comparison of Resource Classification Methodologies With a New Approach"
  Abdullah Arik
- GSLIB, Geostatistical Software Library and User's Guide, 1993
  Clayton V. Deutch, Andre G. Journel

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